Name of College — S. S. College J. Bad

Deptt — Mathematic

Topic — Indeterminate form

$[\frac{0}{0}, \frac{\infty}{\infty}, 1^\infty]$

class — B.Sc I (Hons)

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Problem :→ $\lim_{x\to a} \sqrt{a^2-x^2}\, \cot\left[\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right]$

It is in the form of $0\times\infty$

$$= \lim_{x\to a} \frac{\sqrt{a^2-x^2}}{\tan\left[\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right]} \qquad \left[\frac{0}{0}\right]$$

using L-Hospital Rule.

$$= \lim_{x\to a} \frac{\frac{1}{2\sqrt{a^2-x^2}}(-2x)}{\sec^2\left[\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right]\cdot\frac{\pi}{2}\cdot\frac{1}{2}\sqrt{\frac{a+x}{a-x}}\cdot\frac{-(a+x)-(a-x)}{(a+x)^2}}$$

$$= \lim_{x\to a} \frac{-x}{\frac{\pi}{4}\sec^2\left[\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right](a+x)\cdot\frac{-2a}{(a+x)^2}}$$

$$= \lim_{x\to a} \frac{x(a+x)}{\frac{a\pi}{2}\sec^2\left[\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right]} = \frac{a\cdot 2a}{\frac{a\pi}{2}\cdot 1} = \frac{4a}{\pi}$$

Problem. $\lim_{x\to\infty} 2^x \sin\frac{a}{2^x}$ $[0\times\infty]$

Put $\frac{a}{2^x} = \theta$

$\Rightarrow 2^x = \frac{a}{\theta}$.

Also when $x\to\infty \Rightarrow \theta\to 0$

$\lim_{\theta\to 0} \frac{a}{\theta}.\sin\theta$ $\left[\frac{0}{0}\right]$

using L-Hospital Rule.

$a \lim_{\theta\to 0} \frac{\cos\theta}{1} = a\cos 0 = a$

Problem $\lim_{x\to\infty} 2^x . \tan\frac{a}{2^x}$ $[\infty\times 0]$

Put $\frac{a}{2^x} = \theta \Rightarrow 2^x = \frac{a}{\theta}$.

As $x\to\infty \Rightarrow \theta\to 0$

$\lim_{\theta\to 0} \frac{a}{\theta}.\tan\theta.$ $\left[\frac{0}{0}\right]$

$= a\lim_{\theta\to 0}. \frac{\sec^2\theta}{1}$

$= a.1 = a$

Problem based on the form $\infty - \infty$

Problem $\lim_{x\to\pi/2}[\sec x - \tan x]$

$= \lim_{x\to\pi/2} \frac{1-\sin x}{\cos x}$ $\left[\frac{0}{0}\right]$

$$= \lim_{x \to \pi/2} \frac{-\cos x}{-\sin x}$$ using L-Hospital Rule

$$= \frac{0}{1} = 0$$

Problem $\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\log_e x} \right)$ $[\infty - \infty]$

$$= \lim_{x \to 1} \frac{x \log_e x - x + 1}{(x-1)\log_e x} \quad \left[\frac{0}{0}\right]$$

using L-Hospital Rule.

$$= \lim_{x \to 1} \frac{x \cdot \frac{1}{x} + \log_e x - 1 + 0}{\log_e x + \frac{x-1}{x}}$$

$$= \lim_{x \to 1} \frac{x \log x}{x \log x + x - 1} \quad \left[\frac{0}{0}\right]$$

using again L-Hospital Rule.

$$= \lim_{x \to 1} \frac{\log x + x \cdot \frac{1}{x}}{x \cdot \frac{1}{x} + \log x + 1}$$

$$= \lim_{x \to 1} \frac{\log x + 1}{2 + \log x} = \frac{\log 1 + 1}{2 + \log 1}$$

$$= \frac{1}{2} \quad \because \log 1 = 0$$

Problem :→ Evaluate

$$\lim_{x \to \pi/2} \left(x \tan x - \frac{\pi}{2} \sec x \right) \quad [\infty - \infty]$$

$$= \lim_{x \to \pi/2} \frac{x \sin x}{\cos x} - \frac{\pi}{2 \cos x}$$

$$= \lim_{x \to \pi/2} \frac{2x \sin x - \pi}{2 \cos x} \quad \left[\frac{0}{0}\right]$$

using L-Hospital Rule

$$= \lim_{x \to \pi/2} \frac{2[\sin x + x \cos x] - 0}{-2 \sin x}$$

$$= \frac{\cancel{2}[\sin \pi/2 + \frac{\pi}{2} \cos \frac{\pi}{2}]}{-\cancel{2} \sin \pi/2} = \frac{1+0}{-1} = -1$$

Problem Evaluate

$$\lim_{x \to 0} \frac{1}{x} - \cot x \quad [\infty - \infty]$$

$$= \lim_{x \to 0} \frac{1}{x} - \frac{\cos x}{\sin x}$$

$$= \lim_{x \to 0} \frac{\sin x - x \cos x}{x \sin x} \quad [0/0]$$

using L-Hospital Rule

$$= \lim_{x \to 0} \frac{\cos x - [\cos x + x(-\sin x)]}{\sin x + x \cos x}$$

$$= \lim_{x \to 0} \frac{x \sin x}{\sin x + x \cos x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{\sin x + x \cos x}{\cos x + (\cos x - x \sin x)}$$ Applying L-Hospi Rule again

$$= \lim_{x \to 0} \frac{\sin x + x \cos x}{2 \cos x - x \sin x}$$

$$= \frac{0 + 0}{2 \cos 0 - 0} = \frac{1}{2}$$

Evaluate

$$\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right] \quad [\infty - \infty]$$

$$= \lim_{x \to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x} \quad \left[\frac{0}{0}\right]$$

using L-Hospital Rule

$$= \lim_{x \to 0} \frac{2 \sin x \cos x - 2x}{x^2 \times 2 \sin x \cos x + 2x \sin^2 x}$$

$$= \lim_{x \to 0} \frac{\sin 2x - 2x}{x^2 \sin 2x + 2x \sin^2 x} \quad \left[\frac{0}{0}\right]$$

Again using L-Hospital Rule

$$= \lim_{x \to 0} \frac{2 \cos 2x - 2}{2x^2 \cos 2x + 2x \sin 2x + 2[\sin^2 x + x \times 2 \sin x \cos}$$

$$= \lim_{x \to 0} \frac{2\cos 2x - 2}{2x^2 \cos 2x + 2x \sin 2x + 2\sin^2 x + 2x \sin 2x}$$

$$= \frac{2}{2} \lim_{x \to 0} \frac{\cos 2x - 1}{x^2 \cos 2x + 2x \sin 2x + \sin^2 x} \quad \left[\frac{0}{0}\right]$$

using L-Hospital Rule again

$$= \lim_{x \to 0} \frac{-2\sin 2x}{2x\cos 2x - 2x^2 \sin 2x + 2[\sin 2x + 2x\cos 2x] + 2\sin x \cos x}$$

$$= \lim_{x \to 0} \frac{-2\sin 2x}{6x\cos 2x + 3\sin 2x - 2x^2 \sin 2x} \quad [0/0]$$

using L-Hospital Rule again

$$= -2 \lim_{x \to 0} \frac{-2\cos 2x}{6[\cos 2x - 2x\sin 2x] + 6\cos 2x - 2[2x\sin 2x + 2x^2\cos 2x]}$$

$$= -4 \lim_{x \to 0} \frac{\cos 2x}{6\cos 2x - 12x\sin 2x + 6\cos 2x - 4x\sin 2x - 4x^2\cos 2x}$$

$$= \frac{+4 \times 1}{6 - 0 + 6 - 0 - 0}$$

$$= \frac{4}{12} = 1/3$$

Problem based on the Form
$0^0, \infty^0, 1^\infty$

Problem :→ Evaluate $\lim_{x \to 0} \sin x^{\tan x}$ [Form 0^0]

Let $y = \lim_{x \to 0} \sin x^{\tan x}$

$$\Rightarrow \log y = \lim_{x \to 0} \log \sin x^{\tan x}$$ $\log m^n = n \log m$

$$= \lim_{x \to 0} \tan x \log \sin x$$ $[0 \times \infty]$

$$= \lim_{x \to 0} \frac{\log \sin x}{\cot x}$$ $\left[\frac{\infty}{\infty}\right]$

Applying L-Hospital Rule.

$$= \lim_{x \to 0} \frac{d(\log \sin x)}{d \cot x}$$

$$= -\lim_{x \to 0} \frac{\frac{1}{\sin x} \cdot \cos x}{\operatorname{cosec}^2 x}$$

$$= -\lim_{x \to 0} \frac{\cos x}{\cancel{\sin x} \operatorname{cosec} x}$$

$$= -\lim_{x \to 0} \cos x \sin x$$

$$= -\cos 0 \sin 0 = -0$$

$$\Rightarrow y = e^0 = 1$$

Problem :→ 2.

$$\lim_{x \to 0} x^x \quad [0^0]$$

Let $y = \lim_{x \to 0} x^x$

Taking log of both side.

$$\Rightarrow \log y = \lim_{x \to 0} x \log x \quad [0 \times \infty]$$

$$= \lim_{x \to 0} \frac{\log x}{1/x} \quad \left[\frac{\infty}{\infty}\right]$$

Applying L-Hospital Rule,

$$= \lim_{x \to 0} \frac{d \log x}{d x^{-1}}$$

$$= -\lim_{x \to 0} \frac{x^2}{x} = -\lim_{x \to 0} x = 0$$

$$\therefore \log y = 0 \Rightarrow y = e^0 = 1 \quad \left[\because a^x = N \Leftrightarrow \log_a N = x\right]$$

Evaluate

$$\lim_{x \to \pi/2} [\cos x]^{\cos x} \quad \text{Form } [0^0]$$

Let $y = \lim_{x \to \pi/2} (\cos x)^{\cos x}$

$$\Rightarrow \log y = \lim_{x \to \pi/2} \cos x \log \cos x \quad [0 \times \infty]$$

$$= \lim_{x \to \pi/2} \frac{\log \cos x}{\sec x} \quad [\infty/\infty$$

$$\therefore \log y = \lim_{x \to \pi/2} \frac{\log \cos x}{\sec x} \quad [\infty/\infty]$$

Applying L-Hospital Rule.

$$= \lim_{x \to \pi/2} \frac{\frac{d}{dx} \log \cos x}{\frac{d}{dx} \sec x}$$

$$= \lim_{x \to \pi/2} \frac{\frac{1}{\cos x} (-\sin x)}{\sec x \cdot \tan x}$$

$$= -\lim_{x \to \frac{\pi}{2}} \cos x$$

$$= -\cos \pi/2 = 0$$

$$\therefore y = \cancel{\log e^0}\ e^0 = 1$$

Problem

$$\lim_{x \to \pi/2} [\tan x]^{\cos x} \quad [\infty^0]$$

$$\text{Let } y = \lim_{x \to \pi/2} (\tan x)^{\cos x}$$

$$\Rightarrow \log y = \lim_{x \to \pi/2} \cos x \log \tan x \quad [0 \times \infty]$$

$$= \lim_{x \to \pi/2} \frac{\log \tan x}{\sec x}$$

$$\Rightarrow \log y = \lim_{x \to \pi/2} \frac{\frac{d}{dx} \log \tan x}{\frac{d}{dx} \sec x}$$ [Applying L'Hospital Rule.]

$$= \lim_{x \to \pi/2} \frac{\frac{1}{\tan x} \cdot \sec^2 x}{\sec x \tan x}$$

$$= \lim_{x \to \pi/2} \frac{\sec x}{\tan^2 x}$$

$$= \lim_{x \to \pi/2} \frac{1}{\cos x \times \frac{\sin^2 x}{\cos^2 x}}$$

$$= \lim_{x \to \pi/2} \frac{\cos x}{\sin^2 x} = \frac{0}{1} = 0$$

$$\therefore y = e^0 = 1$$

<u>Evaluate</u> $\lim_{x \to 0} \left[\frac{1}{x}\right]^{\tan x}$ $\left[\text{Form } \infty^0\right]$

Let $y = \lim_{x \to 0} \left(\frac{1}{x}\right)^{\tan x}$

$$\log y = -\lim_{x \to 0} \tan x \log x \quad [0 \times \infty]$$

$$= -\lim_{x \to 0} \frac{\log x}{\cot x} \quad [\infty/\infty]$$

$$= +\lim_{x \to 0} \frac{1}{x \cdot \operatorname{cosec}^2 x}$$ Applying L'Hospital Rule.

$$= \lim_{x \to 0} \frac{\sin^2 x}{x} \quad [0/0]$$

Again Applying L-Hospital Rule

$$\log y = \lim_{x \to 0} \frac{2\sin x \cos x}{1}$$

$$= 0$$

$$\therefore y = e^0 = 1$$

Problem

Evaluate $\lim_{x \to 0} (\cos x)^{\cot x}$ $\left[\text{Form } 1^{\infty}\right]$

Let $y = \lim_{x \to 0} (\cos x)^{\cot x}$

$$\Rightarrow \log y = \lim_{x \to 0} \cot x \log \cos x \quad [\infty \times 0]$$

$$= \lim_{x \to 0} \frac{\log \cos x}{\tan x} \quad \left[\frac{0}{0}\right]$$

Applying L-Hospital Rule.

$$= \lim_{x \to 0} -\frac{\sin x}{\cos x} \cdot \frac{1}{\sec^2 x}$$

$$= -\lim_{x \to 0} \sin x \cos x$$

$$= -0 \times 1 = 0$$

$$\therefore \log y = 0 \Rightarrow y = e^0 = 1$$ ~~proved~~

Problem $\lim_{x\to 0} [\cos x]^{\frac{1}{x^2}}$ $[1^\infty]$

Let $y = \lim_{x\to 0} (\cos x)^{1/x^2}$

$\Rightarrow \log y = \lim_{x\to 0} \frac{\log \cos x}{x^2}$ $[0/0]$

Applying L. Hospital Rule.

$$= \lim_{x\to 0} \frac{\frac{1}{\cos x}(-\sin x)}{2x}$$

$$= -\frac{1}{2}\lim_{x\to 0} \frac{\tan x}{x} \quad [0/0]$$

$$= -\frac{1}{2}\lim_{x\to 0} \frac{\sec^2 x}{1} = -\frac{1}{2}$$

$$\therefore y = e^{-1/2} = \frac{1}{e^{1/2}} = \frac{1}{\sqrt{e}}$$

Problem :→ $\lim_{x\to \pi/2} (\sin x)^{\tan x}$

Let $y = \lim_{x\to \pi/2} (\sin x)^{\tan x}$ $[1^\infty]$

$\Rightarrow \log y = \lim_{x\to \pi/2} \tan x \log \sin x$ $[\infty\times 0]$

$$= \lim_{x\to \pi/2} \frac{\log \sin x}{\cot x} \quad [0/0]$$

Applying L. Hospital Rule

$$= \lim_{x\to \pi/2} \frac{\cot x}{-\operatorname{cosec}^2 x} = -\lim_{x\to \pi/2} \cot x \cdot \sin x$$

$$= -0\times 1 = 0$$

$\therefore y = e^0 = 1.$